National 5 Mathematics

Key Skills Booklet

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine Rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $Area = \dfrac{1}{2}ab \sin C$

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$

Volume of a Pyramid: $V = \frac{1}{3}Ah$

Standard Deviation: $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - \left(\sum x\right)^2 / n}{n-1}}$, where *n* is the sample size

SKILL 1: Compound Percentages

appreciation and depreciation

(1) A £240000 house appreciates in value by 5% in 2007, appreciates 10% in 2008 and depreciates by 15% in 2009. Calculate the value of the house at the end of 2009.

or

evaluate year by year

year 1

5% *increase*: 100% + 5% = 105% = 1·05

5% *of* £240000 = £12000

£240000 + £12000 = £25200

10% *increase*: 100% + 10% = 110% = 1·10

year 2

10% *of* £252000 = £25200

£252000 + £25200 = £277200

15% *decrease*: 100% – 15% = 085% = 0·85

year 3

£240000 × 1·05 × 1·10 × 0·85

15% *of* £277200 = £41580

= £235620

£277200 – £41580 = £235620

compound interest

(2) Calculate the compound interest on £12000 invested at 5% pa for 3 years.

£12000 × $(1·05)^3$ *ie.* × 1·05 × 1·05 × 1·05 *or evaluate year by year*

£12000 × 1·157625

= £13891·50

compound interest = £13891·50 – £12000 = £1891·50

Questions

1 (C) Jack weighs 94 kilograms.

On the 1st of January, he starts a diet which is designed to reduce his weight by 7% per month.

During which month should he achieve his target weight of 73 kilograms?

Show all your working. **4 marks**

2 (C) There are 2·69 million vehicles in Scotland.

It is estimated that this number will increase at a rate of 4% each year.

If this estimate is correct, how many vehicles will there be in 3 years' time?

Give your answer **correct to 3 significant figures.** **4 marks**

3 (C) Olga normally runs a total distance of 28 miles per week.

She decides to increase her distance by 10% a week for the next four weeks.

How many miles will she run in the fourth week? **3 marks**

4 (C) A company buys machinery worth £750 000.

The value of the machinery depreciates by 20% per annum.

The machinery will be replaced at the end of the year in which its value falls below half of its original value.

After how many years should the machinery be replaced?

You must explain your answer. **4 marks**

5 (C) Due to the threat of global warming, scientists recommended in 2010 that the emissions of greenhouse gases should be reduced by 50% by the year 2050.

The government decided to reduce the emissions of greenhouse gases by 15% **every ten years**, starting in the year 2010.

Will the scientists' recommendations have been achieved by 2050?

You must give a reason for your answer.

4 marks

6 (C) It is estimated that house prices will increase at the rate of 3·15% per annum.

A house is valued at £134 750. If its value increases at the predicted rate, calculate its value after 3 years.

Give your answer correct to **four** significant figures.

4 marks

7 (C) It is estimated that an iceberg weighs 84 000 tonnes.

As the iceberg moves into warmer water, its weight decreases by 25% each day.

What will the iceberg weigh after 3 days in the warmer water?

Give your answer **correct to three significant figures.**

4 marks

8 (C) A company makes large bags of crisps which contain 90 grams of fat.

The company aims to reduce the fat content of the crisps by 50%.

They decide to reduce the fat content by 20% each year.

Will they have achieved their aim by the end of the 3rd year?

Justify your answer.

4 marks

9 (C) An industrial machine costs £176 500.

Its value depreciates by 4·25% each year.

How much is it worth after 3 years?

Give your answer correct to **three** significant figures.

4 marks

Answers

1) During April
2) 3.03 million
3) 40.9948 miles
4) 4 years because 307 200 < 375 000
5) No, 0.522 > 0.5
6) £147 900
7) 35 400 tonnes
8) No, because 51.2% > 50%
9) £155 000

SKILL 2: Reverse Percentages

REVERSING PERCENTAGE CHANGE

Divide by the factor which produced the increase.

(1) Including VAT of 20%, a radio costs £96. Find the original cost exclusive of VAT.

20% VAT added

$100\% \xrightarrow{+20\%} 120\% = 1 \cdot 20$

$£x \times 1 \cdot 20 = £96$

$£x = £96 \div 1 \cdot 20$

$= £80$

non-calculator: $120\% \xrightarrow{\div 12} 10\% \xrightarrow{\times 10} 100\%$

$£96 \div 12 = £8 \qquad £8 \times 10 = £80$

(2) A camera costs £120 after a discount of 25% is applied. Find the original cost.

25% discount subtracted

$100\% \xrightarrow{-25\%} 075\% = 0 \cdot 75$

$£x \times 0 \cdot 75 = £120$

$£x = £120 \div 0 \cdot 75$

$= £160$

non-calculator: $75\% = \frac{3}{4} \qquad \frac{3}{4} \xrightarrow{\div 3} \frac{1}{4} \xrightarrow{\times 4} \frac{4}{4}$

$£120 \div 3 = £40 \qquad £40 \times 4 = £160$

Questions

1 (C) Last year, 1296 learner drivers from "Topflight" school of motoring passed their driving test.

This was 72% of those who sat their driving test from Topflight.

How many **failed** their driving test?

3 marks

2 (C) The price for Paul's summer holiday is £894·40.

The price includes a 4% booking fee.

What is the price of his holiday without the booking fee?

3 marks

3 (C) A car is valued at £3780.

This is 16% less than last year's value.

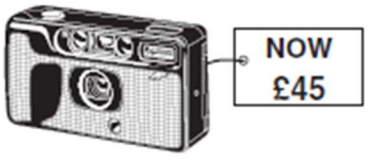
What was the value of the car last year?

3 marks

4 (C) In a sale, all cameras are reduced by 20%.

A camera now costs £45.

Calculate the **original** cost of the camera.



3 marks

5 (NC) Cleano washing powder is on special offer.

Each box on special offer contains 20% more powder than the standard box.

A box on special offer contains 900 grams of powder.

How many grams of powder does the standard box contain?

3 marks

6 (NC) This year, Ben paid £260 for his car insurance.
This is an increase of 30% on last year's payment.

How much did Ben pay last year?

3 marks

7 (C) Mark takes some friends out for a meal.
The restaurant adds a 10% service charge to the price of the meal.
The **total** bill is £148·50.
What was the price of the meal?

3 marks

8 (C) Harry bids successfully for a painting at an auction.
An "auction tax" of 8% is added to his bid price.
He pays £324 in total.
Calculate his bid price.

3 marks

Answers
1) 504
2) £860
3) £4500
4) £56.25
5) 750g
6) £200
7) £135
8) £300

SKILL 3: Simultaneous Equations

SOLVE SIMULTANEOUS EQUATIONS: ELIMINATION METHOD

Can add or subtract multiples of the equations to eliminate either the x or y term.

Solve **algebraically** the system of equations: $4x + 3y = 5$
$5x - 2y = 12$

$4x + 3y = 5$	(1) $\times 2$	*can choose to eliminate x or y term*
$5x - 2y = 12$	(2) $\times 3$	*choosing y term, LCM* (3y,2y) = 6y
$8x + 6y = 10$	(3)	*multiplied each term of* (1) *by* 2 *for* $+6y$
$15x - 6y = 36$	(4)	*multiplied each term of* (2) *by* 3 *for* $-6y$
$23x = 46$	(3) + (4)	*added equations , adding "like" terms*
$x = 2$		$+6y$ *and* $-6y$ *added to* 0 *(ie eliminate)*
$4x + 3y = 5$	(1)	*can choose either equation* (1) *or* (2)
$4 \times 2 + 3y = 5$		*substituted for* $x = 2$
$8 + 3y = 5$		
$3y = -3$		***SOLUTION***: $x = 2$ *and* $y = -1$
$y = -1$		

Questions

1 (NC) Joan buys gold and silver charms to make bracelets.

2 gold charms and 5 silver charms cost £125.

(*a*) Let g pounds be the cost of one gold charm and s pounds be the cost of one silver charm.

Write down an equation in terms of g and s to illustrate the above information. **1 mark**

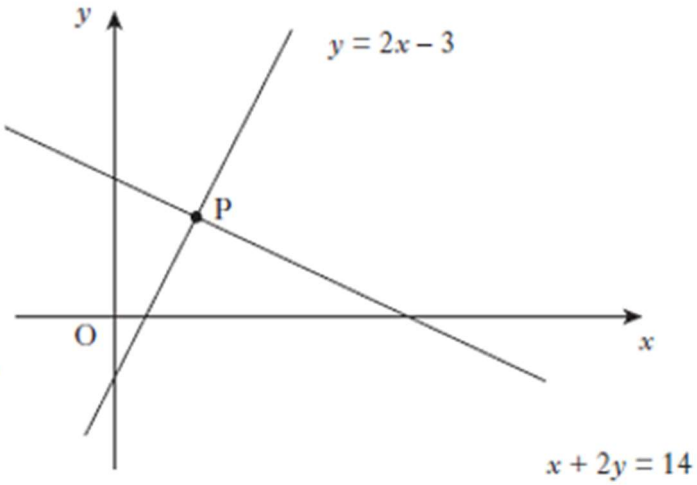
4 gold charms and 3 silver charms cost £145.

(*b*) Write down another equation in terms of g and s to illustrate this information. **1 mark**

(*c*) Hence calculate the cost of each type of charm. **3 marks**

2 (NC) The graph below shows two straight lines.

- $y = 2x - 3$
- $x + 2y = 14$



The lines intersect at the point P.

Find, **algebraically**, the coordinates of P.

4 marks

3 (NC) (*a*) Brian, Molly and their four children visit Waterworld.

The total cost of their tickets is £56.

Let a pounds be the cost of an adult's ticket and c pounds the cost of a child's ticket.

Write down an equation in terms of a and c to illustrate this information. **1 mark**

(*b*) Sarah and her three children visit Waterworld.

The total cost of their tickets is £36.

Write down another equation in terms of a and c to illustrate this information. **1 mark**

(*c*) (i) Calculate the cost of a child's ticket. **2 marks**

(ii) Calculate the cost of an adult's ticket. **1 mark**

4 (C) Three groups are booking a holiday. The first group consists of 6 adults and 2 children. The total cost of their holiday is £3148.

Let x pounds be the cost for an adult and y pounds be the cost for a child.

(*a*) Write down an equation in x and y which satisfies the above information. **1 mark**

The second group books the same holiday for 5 adults and 3 children. The total cost of their holiday is £3022.

(*b*) Write down a second equation in x and y which satisfies this information. **1 mark**

(*c*) The third group books the same holiday for 2 adults and 4 children. The travel agent calculates that the total cost is £2056.

Has this group been overcharged?

Justify your answer. **4 marks**

5(C)

Alan is taking part in a quiz. He is awarded x points for each correct answer and y points for each wrong answer. During the quiz, Alan gets 24 questions correct and 6 wrong. He scores 60 points.

(*a*) Write down an equation in x and y which satisfies the above condition. **1 mark**

Helen also takes part in the quiz. She gets 20 questions correct and 10 wrong. She scores 40 points.

(*b*) Write down a second equation in x and y which satisfies this condition. **1 mark**

(*c*) Calculate the score for David who gets 17 correct and 13 wrong. **4 marks**

Answers

1) g = 25, s = 15
2) (4, 5)
3) Child ticket = £8, Adult ticket = £12
4) Yes – the group has been overcharged by £10
5) 25 points

SKILL 4: Standard Deviation

STANDARD DEVIATION

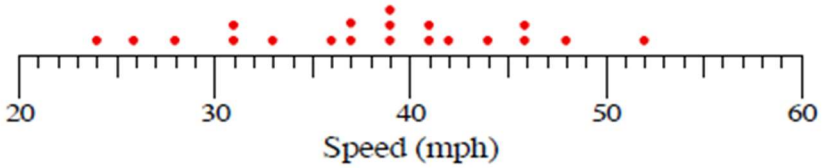
A measure of the spread of a set of data, giving a numerical value to how the data deviates from the mean. It therefore gives an indication of how good the mean is as a representitive of the data set.

Formulae:

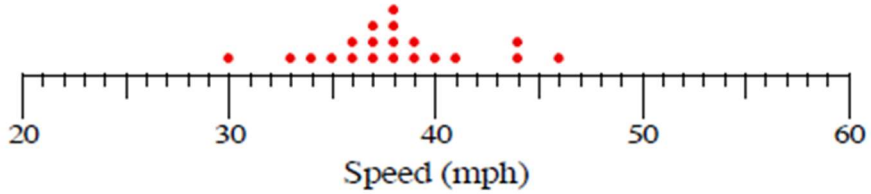
mean $\bar{x} = \dfrac{\sum x}{n}$ standard deviation $s = \sqrt{\dfrac{\sum (x-\bar{x})^2}{n-1}}$ or $s = \sqrt{\dfrac{\sum x^2 - \dfrac{(\sum x)^2}{n}}{n-1}}$

Examples,

(1) High Standard Deviation: results spread out mean = 38 , standard deviation = 7·5



(2) Low Standard Deviation: results clustered around the mean
the results are more consistent mean = 38 , standard deviation = 3·8



The pulse rates of 8 army recruits: 61, 64, 65, 67, 70, 72, 75, 78 beats per minute.

$\bar{x} = \dfrac{\sum x}{n}$

$= \dfrac{552}{8}$

$= 69$

x	$x-\bar{x}$	$(x-\bar{x})^2$
61	-8	64
64	-5	25
65	-4	16
67	-2	4
70	+1	1
72	+3	9
75	+6	36
78	+9	81
TOTALS 552		236

$s = \sqrt{\dfrac{\sum (x-\bar{x})^2}{n-1}}$

$= \sqrt{\dfrac{236}{7}}$

$= 5 \cdot 806....$

$\approx 5 \cdot 8$

Questions

1 (C) Harry often plays golf and the scores for some of his games are recorded below.

84 78 87 80 81

(*a*) For this sample calculate:

(i) the mean; **1 mark**

(ii) the standard deviation. **3 marks**

Show clearly all your working.

(*b*) His partner for these games is Tony, whose scores are listed below.

104 98 107 100 101 **2 marks**

Write down the mean and standard deviation of Tony's scores.

2 (C) A ten-pin bowling team recorded the following six scores in a match.

134 102 127 98 104 131

(*a*) For this sample calculate:

(i) the mean;

(ii) the standard deviation.

Show clearly all your working. **4 marks**

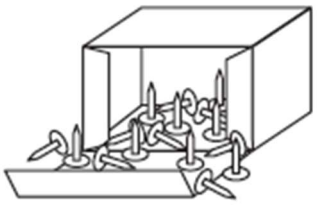
In their second match their six scores have a mean of 116 and a standard deviation of 12·2.

(*b*) Consider the 5 statements written below.

1 The total of the scores is the same in both matches.
2 The total of the scores is greater in the first match.
3 The total of the scores is greater in the second match.
4 In the first match the scores are more spread out.
5 In the second match the scores are more spread out.

Which of these statements is/are true? **2 marks**

3 (C) A sample of six boxes contains the following numbers of pins per box.



43 39 41 40 39 44

(*a*) For the above data, calculate:

(i) the mean; **1 mark**

(ii) the standard deviation. **3 marks**

The company which produces the pins claims that "the mean number of pins per box is 40 ± 2 and the standard deviation is less than 3".

(*b*) Does the data in part (*a*) support the claim made by the company?

Give reasons for your answer. **2 marks**

4 (C) A machine is used to put drawing pins into boxes.

A sample of 8 boxes is taken and the number of drawing pins in each is counted.

The results are shown below:

102 102 101 98 99 101 103 102

(*a*) Calculate the mean and standard deviation of this sample. **3 marks**

(*b*) A sample of 8 boxes is taken from another machine.

This sample has a mean of 103 and a standard deviation of 2·1.

Write down two valid comparisons between the samples. **2 marks**

Answers

1) (a) 82 (b) s = 3.54 (c) Mean = 102, s = 3.54
2) (a) (i) 116 (ii) 16.33 (b) 1 and 4
3) (a) 41 (b) s = 2.1 (c) Yes, with two statements comparing values
4) (a) mean = 101, s = 1.69 (b) comparison involving 'average' and 'spread/consistency

SKILL 5: Solving Equations

WITH BRACKETS remove first and simplify

(2) Solve: $(4x+3)(x-2)=(2x-3)^2$

$$4x^2-5x-6=4x^2-12x+9 \quad \textit{removed brackets, fully simplifying}$$
$$-5x-6=-12x+9 \quad \textit{subtracted } 4x^2 \textit{ from each side}$$
$$7x-6=+9 \quad \textit{added } 12x \textit{ to each side}$$
$$7x=15 \quad \textit{added } 6 \textit{ to each side}$$
$$x=\tfrac{15}{7} \quad \textit{divided each side by } 7$$

WITH FRACTIONS remove first, multiplying by the LCM of the denominators

(3) Solve $\frac{1}{2}(x+3)+\frac{1}{3}x=1$

$$\tfrac{3}{6}(x+3)+\tfrac{2}{6}x=\tfrac{6}{6} \quad \textit{write with common denominators}$$
$$3(x+3)+2x=6 \quad \textit{both sides} \times 6 \textit{ to remove fractions}$$
$$3x+9+2x=6$$
$$5x=-3$$
$$x=-\tfrac{3}{5}$$

Questions

1 (NC) Solve the equation

$$3x+1=\frac{x-5}{2}.$$

3 marks

2 (NC) Solve the equation

$$x-2(x+1)=8.$$

3 marks

3 (NC) Solve the equation

$$\frac{2}{x}+1=6.$$

3 marks

4(NC) Solve **algebraically** the equation

$$2x-\frac{(3x-1)}{4}=4.$$

3 marks

5(NC) Solve the equation

$$\frac{2x}{3}-\frac{5}{6}=2x.$$

3 marks

Give your answer in its simplest form.

Answers

1) $x = -{}^{7}/_{5}$ 2) $x = -10$ 3) $x = {}^{2}/_{5}$ 4) $x = 3$ 5) $x = -{}^{5}/_{8}$

SKILL 6: Multiplying Brackets

DOUBLE BRACKETS

(1) $(3x+2)(2x-5)$

$= 3x(2x-5) + 2(2x-5)$

$= 6x^2 - 15x + 4x - 10$

$= 6x^2 - 11x - 10$

or

"FOIL"

$(3x+2)(2x-5)$

(2) $(2t-3)^2$

$= (2t-3)(2t-3)$

$= 2t(2t-3) - 3(2t-3)$

$= 4t^2 - 6t - 6t + 9$

$= 4t^2 - 12t + 9$

(3) $(w+2)(w^2-3w+5)$

$= w(w^2-3w+5) + 2(w^2-3w+5)$

$= w^3 - 3w^2 + 5w + 2w^2 - 6w + 10$

$= w^3 - 3w^2 + 2w^2 + 5w - 6w + 10$

$= w^3 - w^2 - w + 10$

Questions

1 (NC) Expand and simplify

$(2x-5)(x^2+3x-7)$. **3 marks**

2 (NC) Expand and simplify

$(3x-2)(2x^2+x+5)$. **3 marks**

3 (C) Expand and simplify

$(3x+1)(x^2-5x+4)$. **3 marks**

4 (C) Multiply out the brackets and collect like terms.

$(x+2)(x-5)-9x$ **3 marks**

5 (C) Multiply out the brackets and collect like terms.

$(3x-5)(x^2+2x-6)$ **3 marks**

6 (NC) Multiply out the brackets and collect like terms.

$5x+(3x+2)(2x-7)$ **3 marks**

7 (C) Expand fully and simplify

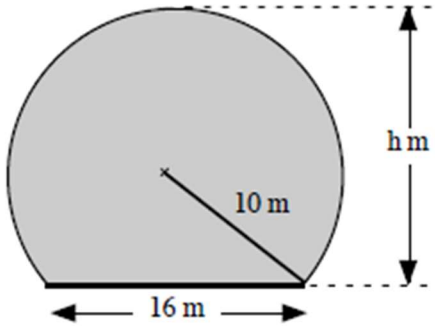
$x(x-1)^2$. **2 marks**

Answers

1) $2x^3 + x^2 - 29x + 35$
2) $6x^3 - x^2 + 13x - 10$
3) $3x^3 - 14x^2 + 7x + 4$
4) $x^2 - 12x - 10$
5) $3x^3 + x^2 - 28x + 30$
6) $6x^2 - 12x - 14$
7) $x^3 - 2x^2 + x$

SKILL 7: Pythagoras in a Circle

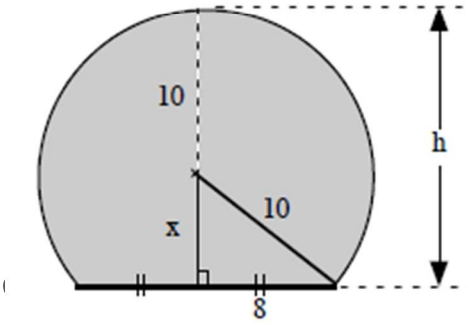
PYTHAGORAS' THEOREM



A circular road tunnel, radius 10 metres, is cut through a hill.

The road has a width 16 metres.

Find the height of the tunnel.



the diameter drawn is the perpendicular bisector of the chord:
Δ is right-angled so can apply Pyth. Thm.

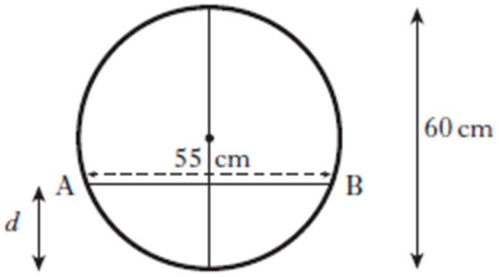
$x^2 = 10^2 - 8^2$
$= 100 - 64$
$= 36$
$x = \sqrt{36}$
$x = 6$

$h = x + 10$
$= 6 + 10$
$h = 16$

height 16 metres

1 (C)

Water flows through a horizontal pipe of diameter 60 centimetres.
The surface width, AB, of the water is 55 centimetres.



4 marks

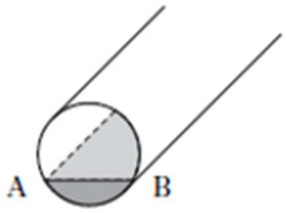
(*a*) Calculate the depth, *d*, of the water in the pipe.

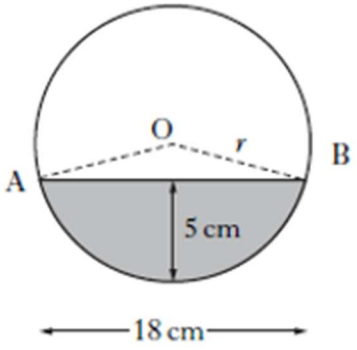
1 mark

(*b*) What other depth of water would give the same surface width?

2 (C)

A pipe has water in it as shown.

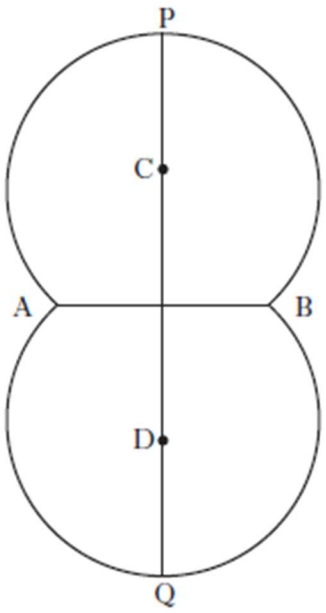




The depth of the water is 5 centimetres.
The width of the water surface, AB, is 18 centimetres.
Calculate *r*, the radius of the pipe.

4 marks

3 (C) The shape below is used as a logo in an advertising campaign. It is made up from segments of two identical circles.



The points C and D are the centres of the circles and each circle has a radius of 24 centimetres.

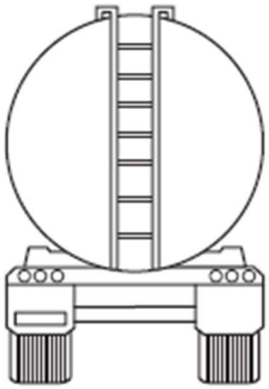
AB is a common chord of length 30 centimetres.

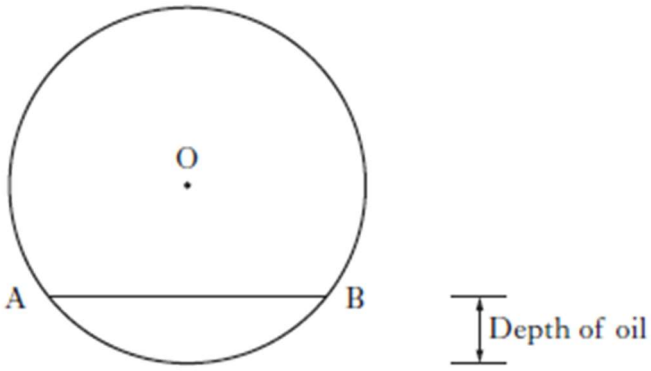
Calculate the height of the logo, represented by the line PQ.

5 marks

4 (C) A tanker delivers oil to garages.

The tank has a circular cross-section as shown in the diagram below.





The radius of the circle, centre O, is 1·9 metres.

The width of the surface of the oil, represented by AB in the diagram, is 2·2 metres.

Calculate the depth of the oil in the tanker.

4 marks

Answers

1) (a) 18.01cm (b) 41.99cm
2) 10.6cm
3) 85.4cm
4) 0.4m

SKILL 8: Equation of a Straight Line

EQUATION OF A STRAIGHT LINE

gradient m , y-intercept C units ie. meets the y-axis at $(0,C)$ $\quad y = mx + C$

gradient m , through the point (a,b) $\quad y - b = m(x - a)$

equation of line RS:

$m_{RS} = -\frac{3}{2}$ $\quad y = mx + C$

$R(0,8)$ $\quad C = 8$ $\quad y = -\frac{3}{2}x + 8$

equation of line PQ:

$m_{PQ} = \frac{2}{3}$ $\quad y - b = m(x - a)$

a b

$Q(6, 1)$ $\quad y - 1 = \frac{2}{3}(x - 6)$

or use point P

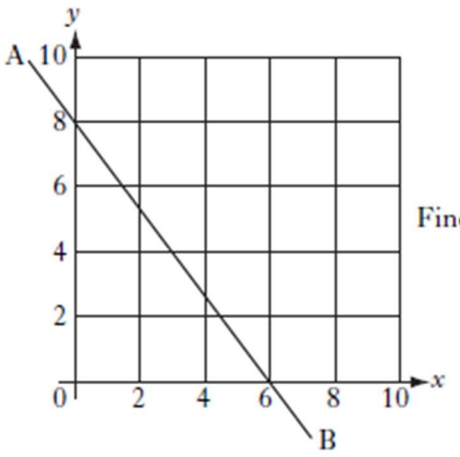
$3y - 3 = 2(x - 6)$

$3y - 3 = 2x - 12$

$3y = 2x - 9$

Questions

1 (NC)



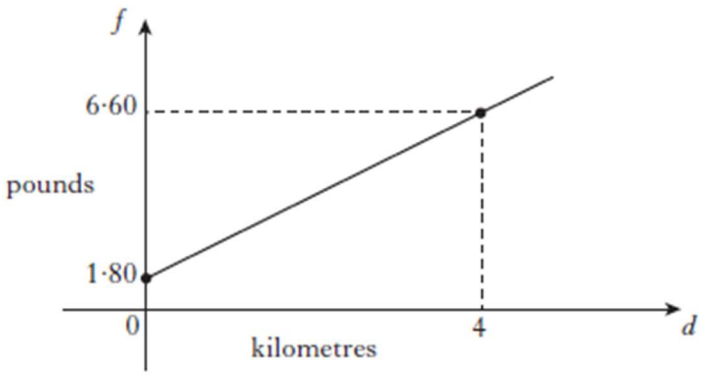
Find the equation of the straight line AB shown in the diagram.

3 marks

2 (C) A taxi fare consists of a call-out charge of £1·80 **plus** a fixed cost per kilometre.

A journey of 4 kilometres costs £6·60.

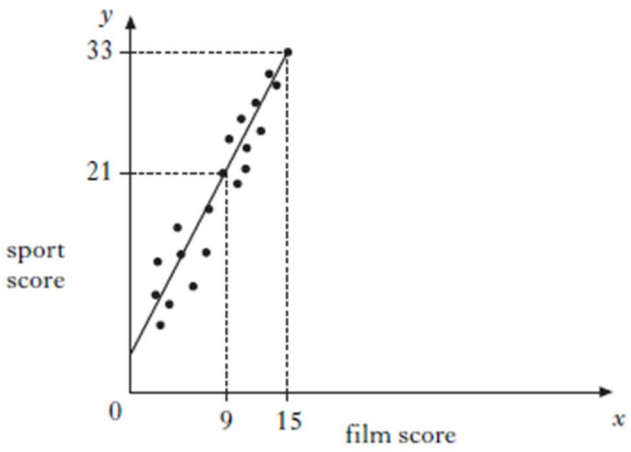
The straight line graph shows the fare, f pounds, for a journey of d kilometres.



(*a*) Find the equation of the straight line. **3 marks**

(*b*) Calculate the fare for a journey of 7 kilometres. **2 marks**

3 (C) Teams in a quiz answer questions on film and sport.
This scatter graph shows the scores of some of the teams.

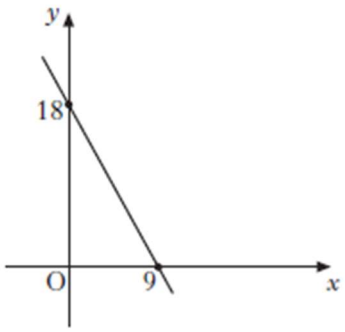


A line of best fit is drawn as shown above.

(*a*) Find the equation of this straight line. **2 marks**

(*b*) Use this equation to estimate the sport score for a team with a film score of 20. **4 marks**

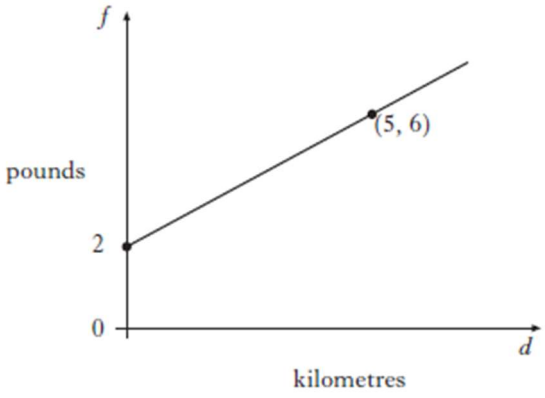
4 (NC) A straight line cuts the x-axis at the point $(9, 0)$ and the y-axis at the point $(0, 18)$ as shown.



Find the equation of this line. **3 marks**

5 (NC) A taxi fare consists of a £2 "call-out" charge **plus** a fixed amount per kilometre.

The graph shows the fare, f pounds for a journey of d kilometres.



The taxi fare for a 5 kilometre journey is £6.

Find the equation of the straight line in terms of d and f. **4 marks**

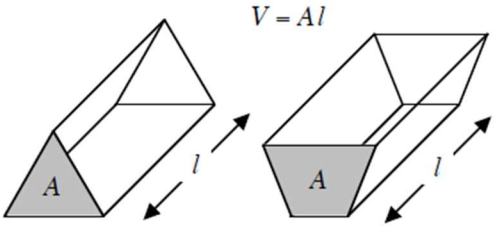
Answers

1) $y = -\frac{4}{3}x + 8$ 2) (a) $f = 1.2d + 1.8$ (b) £10.20 3) (a) $y = 2x + 3$ (b) 43

4) $y = -2x + 18$ 5) $f = 0.8d + 2$

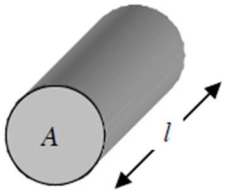
SKILL 9: Formula Backwards

PRISMS a solid with the same cross-section throughout its length.
length l is at right-angles to the area A.

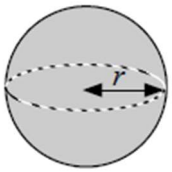


$V = Al$

cylinder $V = \pi r^2 h$

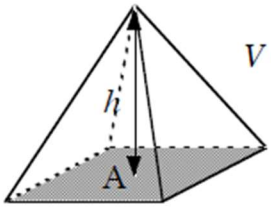


SPHERE

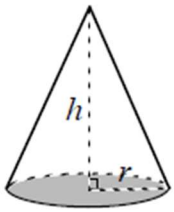


$V = \frac{4}{3}\pi r^3$

PYRAMIDS



$V = \frac{1}{3}Ah$

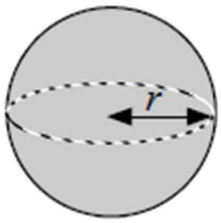


cone

$V = \frac{1}{3}\pi r^2 h$

WORKING BACKWARDS

This sphere has a volume of 2500cm^3. What is its radius?



$$V = \frac{4}{3}\pi r^3$$

$$2500 = \left(\frac{4}{3}\pi\right) \times r^3$$

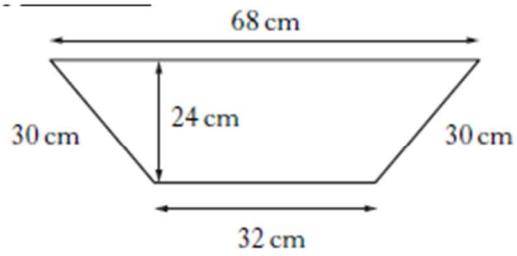
$$\frac{2500}{\left(\frac{4}{3}\pi\right)} = r^3$$

$$r^3 = 596.831\ldots$$

$$r = \sqrt[3]{596.831\ldots}$$

Questions

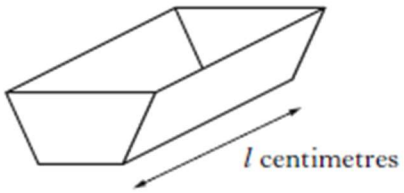
1 (C) A flower planter is in the shape of a prism.
The cross-section is a trapezium with dimensions as shown.



(*a*) Calculate the area of the cross-section of the planter.

(*b*) The volume of the planter is 156 litres.

2 marks



Calculate the length, l centimetres, of the planter. **3 marks**

2 (C) A lead **cube**, of side 10 centimetres, is melted down.
During this process 8% of the metal is lost.
The remaining metal is then made into a **cone**, with radius 8 centimetres.
Calculate the height of this cone.
Give your answer correct to 2 significant figures. **5 marks**

3 (C) The Battle of Largs in 1263 is commemorated by a monument known as The Pencil.

This monument is in the shape of a cylinder with a cone on top.

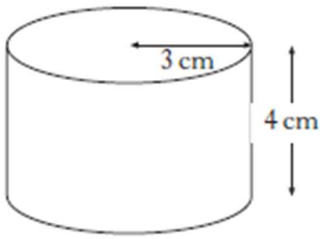
The cylinder part has diameter 3 metres and height 15 metres.

(*a*) Calculate the volume of the **cylinder** part of The Pencil. **2 marks**

The volume of the **cone** part of The Pencil is 5·7 cubic metres.

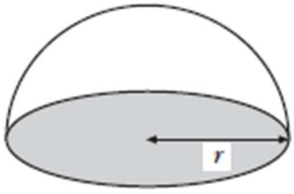
(*b*) Calculate the **total** height of The Pencil. **3 marks**

4 (C) (*a*) A cylindrical paperweight of radius 3 centimetres and height 4 centimetres is filled with sand.



Calculate the volume of sand in the paperweight. **2 marks**

(*b*) Another paperweight, in the shape of a hemisphere, is filled with sand.



It contains the same volume of sand as the first paperweight.

Calculate the radius of the hemisphere.

[The volume of a hemisphere with radius r is given by the formula, $V = \frac{2}{3}\pi r^3$]. **3 marks**

Answers

1) (a) 1200cm^2 (b) 130cm
2) 14cm
3) (a) 106m^3 (b) 17.4m
4) (a) 113.1cm^3 (b) 3.78cm

SKILL 10: Quadratic Formula

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad , a \neq 0$$

Find the roots of the equation $3x^2 - 5x - 1 = 0$, correct to two decimal places.

$3x^2 - 5x - 1 = 0$
$ax^2 + bx + c = 0$

$a = 3 \ , \ b = -5 \ , \ c = -1$

$b^2 - 4ac = (-5)^2 - 4 \times 3 \times (-1) = 37$

$-b = -(-5) = +5$
$2a = 2 \times 3 = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{37}}{6}$$

$$= \frac{5 - \sqrt{37}}{6} \quad or \quad \frac{5 + \sqrt{37}}{6}$$

$$= \frac{-1 \cdot 0827....}{6} \quad or \quad \frac{11 \cdot 0827....}{6}$$

$$x = -0 \cdot 1804.... \quad or \quad 1 \cdot 8471....$$

roots are $-0 \cdot 18$ and $1 \cdot 85$

Questions

1 (C) Solve the equation

$$2x^2 + 7x - 3 = 0.$$

Give your answers **correct to 1 decimal place.** **4 marks**

2 (NC) Given $2x^2 - 2x - 1 = 0$, show that

$$x = \frac{1 \pm \sqrt{3}}{2}$$ **4 marks**

3 (C) Solve the equation

$$2x^2 + 3x - 7 = 0.$$

Give your answers **correct to 2 significant figures.** **4 marks**

4 (C) Solve the equation

$$x^2 - 5x - 2 = 0,$$

giving the roots correct to one decimal place. **4 marks**

5 (C) Solve the equation

$$3x^2 + 7x - 5 = 0,$$

giving the roots correct to one decimal place. **4 marks**

Answers

1) x = 0·4 or −3·9 2) proof 3) x = -2.8 or 1.3 4) x = –0·4 or 5·4 5) x = –2·9 or 0·6